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Surname	
Forename(s)	ANSWERS
Candidate signature	

# A-level **MATHEMATICS**

Unit Pure Core 4

Friday 17 June 2016

Afternoon

Time allowed: 1 hour 30 minutes

### **Materials**

For this paper you must have:

the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



## Answer all questions.

Answer each question in the space provided for that question.

1 (a) Express 
$$\frac{19x-3}{(1+2x)(3-4x)}$$
 in the form  $\frac{A}{1+2x} + \frac{B}{3-4x}$ .

[3 marks]

- (b) (i) Find the binomial expansion of  $\frac{19x-3}{(1+2x)(3-4x)}$  up to and including the term in  $x^2$ . [7 marks]
  - (ii) State the range of values of x for which this expansion is valid.

[1 mark]

QUESTION PART REFERENCE	Answer space for question 1
1a)	19x - 3 = A(3-4x) + B(1+2x)
	$x = \frac{3}{4} - \frac{19(\frac{3}{4}) - 3}{3} = B(1 + 2(\frac{3}{4}))$
	$\frac{57/4-3=10/4B}{45/4=10/4B}$
	B=9/2 OR 4.5
	$x = -1/2 \rightarrow 19(-1/2) - 3 = A(3 - 4(-1/2))$
	$\frac{-19}{2} - 3 = 5A$ $\frac{-25}{2} = 5A$
	A = - 5/2 OR - 2-5
	19x - 3 = -2.5 + 4.5
	$(1+2\pi)(3-4\pi)$ $(1+2\pi)$ $(3-4\pi)$
	$=$ -5 + 9 $\frac{2(1+1x)}{2(3-4x)}$
	2(1+12) 2(3-4)2)

JESTION PART FERENCE	Answer space for question 1
bi)	$\frac{-5(1+7x)^{-1}+9(3-4x)^{-1}}{2}$
	$\frac{-5(1+7x)^{-1}}{2} = -5(1+(-1)(7x)+(-1)(-1)(1x)^{2}$
	$\frac{1}{2} = -\frac{5}{2} \left( 1 - 2\chi + 4\chi^2 \right)$
	$\frac{9(3-4x)^{-1} = \frac{9}{2}(3)^{-1}(1-\frac{4}{3}x)^{-1}}{2}$
	$=\frac{3}{2}\left[1+(-1)(-\frac{4}{3}x)+(-1)(-2)(-\frac{4}{3})^{2}\right]$
	$= \frac{3}{2} \left( 1 + \frac{4}{3} x + \frac{16}{9} x^{2} \right)$
	$\frac{-S(1-1x+4x^2)+\frac{3}{2}(1+\frac{4}{3}x+\frac{16}{9}x^2)}{2}$
	$\frac{-5 + 5x - 10x^{2} + 3 + 7x + 8x^{2}}{2}$
	-1 + 7x - 22 x2
ii)	valid for 12x1<1 OR 1-4/3x1<1
	12/(12) $12/(3/4)$
	: 1/2 < x < 1/2
	Turn over

2 By forming and solving a suitable quadratic equation, find the solutions of the equation

$$3\cos 2\theta - 5\cos \theta + 2 = 0$$

in the interval  $0^{\circ} < \theta < 360^{\circ}$  , giving your answers to the nearest  $0.1^{\circ}.$ 

[5 marks]

QUESTION PART REFERENCE	Answer space for question 2
2)	$3\cos 2\theta - 5\cos \theta + 2 = 0$ $0 < \theta < 360^{\circ}$ . $\cos 2\theta = 2\cos^{2}\theta - 1$
	$\left[\cos 2\theta = 2\cos^2\theta - 1\right]$
	3 (2cos20-1) - Scos 0 + 2 = 0
	$6(0)^{2}\theta - 3 - 5(0)\theta + 2 = 0$
	600120 - 50010 -1=0
	$(6\cos\theta + 1)(\cos\theta - 1) = 0$
	6(0)A+1=0 OR COIA-1=0
	$\cos \theta = -1/6 \qquad \cos \theta = 1$
	θ = 99.59406, ' Θ = 0' -1 out of 260.4059
	260.4059 ruge
	5 A
	180-80.4
	80.4
	7
	270
	D=99-6°, 260.4° (newest 0-1°)

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QUESTION PART REFERENCE	Answer space for question 2
REFERENCE	
	a .
-	
	<b>A</b> <sub>3</sub>



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3 (a) Express 
$$\frac{3+13x-6x^2}{2x-3}$$
 in the form  $Ax + B + \frac{C}{2x-3}$ .

[4 marks]

(b) Show that 
$$\int_3^6 \frac{3+13x-6x^2}{2x-3} dx = p+q \ln 3$$
, where  $p$  and  $q$  are rational numbers.

[4 marks]

0.15051011	
QUESTION PART REFERENCE	Answer space for question 3
3a)	$3 + 13x - 6x^2 \rightarrow (-3x + 2)$
	$2x-3$ $2x-3 - 6x^2 + 13x + 3$
	$-6x^2+9x$
	0 + 4x + 5
	0 + 4x + 3 $-4x - 6$
	0(+9)
	$3 + 13x - 6x^2 = -3x + 2 + 9$
	2x-3 $2x-3$
	A = -3, $B = 2$ , $C = 9$
6)	$\int_{0}^{6} -3x + 2 + 9 dx$
	2 2
-	$\frac{1-3\lambda}{2}$ $\frac{7}{2}$ $\frac{1}{2}$
	$\left(-3(6)^{2}+2(6)+\frac{9}{2}\ln(2(6)-3)\right)-\left(-3(3)^{2}+2(3)+\frac{9}{2}\ln(2(3)-3)\right)$
	$\begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 $
	(-1/2 + 9/09) - (-15 + 9/03)
	$\left(-42 + \frac{9}{2} \ln 9\right) - \left(-15 + \frac{9}{2} \ln 3\right)$
	$\frac{-69 + 9 /n 3^2 - 9 /n 3}{2}$
	2 2
	$-\frac{69}{2} + \frac{18}{2} \ln 3 - \frac{9}{2} \ln 3 = -\frac{69}{2} + \frac{9}{2} \ln 3$



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Answer space for question 3	OUESTION	Amount
	PART	Answer space for question 3
	REFERENCE	
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4 The mass of radioactive atoms in a substance can be modelled by the equation

$$m = m_0 k^t$$

where  $m_0$  grams is the initial mass, m grams is the mass after t days and k is a constant. The value of k differs from one substance to another.

(a) (i) A sample of radioactive iodine reduced in mass from 24 grams to 12 grams in 8 days. Show that the value of the constant k for this substance is 0.917004, correct to six decimal places.

[1 mark]

- (ii) A similar sample of radioactive iodine reduced in mass to 1 gram after 60 days.Calculate the initial mass of this sample, giving your answer to the nearest gram.[2 marks]
- (b) The half-life of a radioactive substance is the time it takes for a mass of  $m_0$  to reduce to a mass of  $\frac{1}{2}m_0$ .

A sample of radioactive vanadium reduced in mass from exactly  $10~{\rm grams}$  to  $8.106~{\rm grams}$  in  $100~{\rm days}$ .

Find the half-life of radioactive vanadium, giving your answer to the nearest day.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 4
NEI ENEROL	
4ai)	$M = M_0 k^t$
	Mo = 24, M=12, 1=8
	12 = 24 X K 8
	1 = k8
	$\frac{1}{k} = k^{8}$ $k = \sqrt[6]{1/2} = 0.9170040. = -0.917004(6dp)$
· i)	M = Mo x 0.917004 t M=1, t=60
	1 = Mo X 0.917004 60
	Me = 1 = 181.0198477
	0.91700460 = 18/g (newest g)



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QUEST	
PAR REFERE	Allswer space for question 4
, ~	
_6	110 10, M 8:106 F=100
	8.106 = 10 x k 100
	$k = \frac{100}{8.106} = 0.9979023$
	10
	M=5, Mo=10, K=0.9979023.
	71 3/10 TO X - 0 - 11 1 10 2 3
	S = 10 x 0.99 79023. E
	0.5 = 0.9979023.
	log 0.5 = t log 0.9979023.
	t = 10g0.5
	log 0.9979023.
	t = 330.08 <b>529</b> 28.
	t= 330 days (neasest day)
	= 3 day)
31	
18111 <b>BB</b> 110 11	



- It is given that  $\sin A = \frac{\sqrt{5}}{3}$  and  $\sin B = \frac{1}{\sqrt{5}}$ , where the angles A and B are both acute.
  - (a) (i) Show that the exact value of  $\cos B = \frac{2}{\sqrt{5}}$ .

[1 mark]

(ii) Hence show that the exact value of  $\sin 2B$  is  $\frac{4}{5}$ .

[2 marks]

(b) (i) Show that the exact value of  $\sin(A-B)$  can be written as  $p(5-\sqrt{5})$ , where p is a rational number.

[4 marks]

(ii) Find the exact value of  $\cos(A-B)$  in the form  $r+s\sqrt{5}$ , where r and s are rational numbers.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 5	
5)	$\int S = \chi = \int (SS)^2 - I^2$	OR sin^28 * + (0)^8 = 1
ai)	$\frac{h}{\chi(z)} = \frac{\chi \cdot 2}{\chi(z)}$	$\left(\frac{1}{\sqrt{5}}\right)^2 + \left(os^2 B^2 = 1\right)$
	$GSB = \frac{2}{5} (as reg)$	$\frac{1}{5} + \cos^2 B^2 = 1$ $\cos^2 B = 4/5$
_ii)	SIN 28 = 2 SIN B COS B	$GJR = \boxed{4 = 2}$
	$\frac{2}{\sqrt{5}}\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$	
	= 4 (as reg)	
	£	



QUESTION	Answer space for question 5
PART REFERENCE	anewer space for question 5
bi)	$\cos A = 1 - (\frac{55}{3})^2 = 1 - \frac{5}{4} = \frac{4}{9} = \frac{2}{13}$
	(3)
	$\sin A = \sqrt{5}$
-	$\sin A = \sqrt{3}$ $\sin B = \frac{1}{\sqrt{3}}$ $\cos A = \frac{2}{3}$ $\cos B = \frac{2}{\sqrt{5}}$
	V
	sin (A-B) = sin A COIB - COIASIN B
	$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{\sqrt{5}}\right) - \left(\frac{2}{3}\right)\left(\frac{1}{\sqrt{5}}\right)$
	= 25s - 2 = 25s - 2 x 35s
	355 355 355
	= 6(5)-655
	9(5)
-	= 30 - 655 = 6(5 - 55)
	45 45
	· = 2 (5 (5)
	$=\frac{2}{15}\left(5-55\right)$
	p=7/15 /5
)	(a, b, b, c,
-11)	COS(A-B) = COSACOSB + SINASINB
	$= \left(\frac{2}{3}\right)\left(\frac{2}{5}\right) + \left(\frac{5}{3}\right)\left(\frac{1}{5}\right)$
	- 4 5
	= 4 + JS = 4+JS x 3JS 3JS 3JS 3JS 3JS
	= 12,5 + 3(5) = 12,5 + 15
	$= 12\sqrt{5} + 3(5) = 12\sqrt{5} + 15$ $9(5)$ $45$
	$\frac{=455+1}{15} OR \frac{1}{3} + 455$
	r=1/3, s=4/15



The line  $l_1$  passes through the point A(0, 6, 9) and the point B(4, -6, -11).

The line  $l_2$  has equation  $\mathbf{r} = \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$ .

(a) The acute angle between the lines  $l_1$  and  $l_2$  is  $\theta$ .

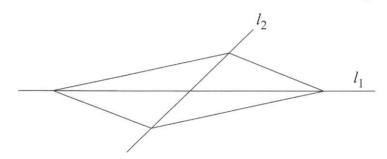
Find the value of  $\cos\theta$  as a fraction in its lowest terms.

[5 marks]

(b) Show that the lines  $l_1$  and  $l_2$  intersect and find the coordinates of the point of intersection.

[5 marks]

(c) The points C and D lie on line  $l_2$  such that ACBD is a parallelogram.



The length of AB is three times the length of CD.

Find the coordinates of the points C and Q.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 6
6a)	$a = \begin{pmatrix} 4 - 0 \\ -6 - 6 \\ -11 - 9 \end{pmatrix} = \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix}$ $b = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$
	a.b = 4 x 3 + -12 x -5 + -20 x l
	= 12 + 60 - 20 = (52)
	$ a  = \sqrt{(4)^2 + (-12)^2 + (-20)^2} = 4\sqrt{35}$
	$ b  = \sqrt{(3)^2 + (-5)^2 + (1)^2} = \sqrt{35}$
	1a1/61 = 4 535 x 535 = (140)
	$\cos \theta = a.b = 52 = 13$
	1a1161 140 35



OUESTION Answer space for question 6
PART REFERENCE
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$\left(\begin{array}{c c} q \end{array}\right) \left(\begin{array}{c} -20 \end{array}\right) \left(\begin{array}{c} -2 \end{array}\right) \left(\begin{array}{c} -1 \end{array}\right)$
0 + 4 m = -1 + 3 x 0
6-12M = 5-5L 2
$9 20\mu = -2 + 2$ 3
using () and (2) - 4 ps - 3 L = -1 (1)(x3)
-12m +5L = -1 @
+ 12m - 9 x = -3
- 4 L = - 4
Z = 1
$4\mu - 3(1) = -1$
4 <sub>M</sub> = 2
m' = 1/2
check in 3 -> 9-20(1/2) =-2+1
9-10 = -2+1
: intersect
when L=1 -1 (-1) + 1/3
when $\lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$
= (2) = (2 0 -1)
$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$
(-1)
check, when $\mu^{=1/2} \rightarrow \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$
19/ (-20/ (-1)
Turn over N



QUESTION PART REFERENCE	Answer space for question 6
c)	length AB (1,) = $\int 4^2 + 12^2 + 20^2 = 4\sqrt{35}$ or $\sqrt{560}$ length CD (12) = $\int \sqrt{560}$ OR $\int \sqrt{35}$
	Tergth CD (12) = 1 J560 OR 4 J35
	midpoint of CD is (2,0,-1) (M)
	1 length CM = 2 535
	$\frac{10  \text{GeV}}{3}$
	$C = \begin{pmatrix} -1 + 3\lambda \\ S + -5\lambda \end{pmatrix} \qquad M = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$
	$\begin{pmatrix} S + - S \lambda \\ -2 + \lambda \end{pmatrix}$
	$CM = \sqrt{(-1+3\lambda-2)^2 + (s-s\lambda)^2 + (-2+\lambda+1)^2}$
	$\frac{2}{3}\sqrt{35} = \sqrt{(-3+3/)^2 + (5-5/)^2 + (-1+/)^2}$
	$\frac{2}{3}\sqrt{35} = \sqrt{9 - 18\lambda + 9\lambda^2 + 25 + 50\lambda + 15\lambda^2 + 1 - 2\lambda + \lambda^2}$
	$\frac{2}{3}\sqrt{35} = \sqrt{35}\lambda^2 - 70\lambda + 35$ (2)
	352² - 702 + 35 = 140 9
	9
	31522 - 6302+315=140
	31522-6302+175=0 (=35)
	$9 \lambda^2 - 18 \lambda + 5 = 0$
	$(3\lambda - 1)(3\lambda - 5) = 0$
	$L = \frac{1}{3}$ , $L = \frac{5}{3}$ When $L = \frac{1}{3}$ : — When $L = \frac{5}{3}$ : —
	When $L = \frac{1}{3} : -$ $C = (0, \frac{10}{3}, \frac{5}{3})$ $D = (4, \frac{-10}{3}, \frac{-1}{3})$



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QUESTION PART REFERENCE	Answer space for question 6
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7 A curve C is defined by the parametric equations

$$x = \frac{4 - e^{2 - 6t}}{4}, \quad y = \frac{e^{3t}}{3t}, \quad t \neq 0$$

(a) Find the exact value of  $\frac{dy}{dx}$  at the point on C where  $t = \frac{2}{3}$ .

[5 marks]

- (b) Show that  $x = \frac{4 e^{2 6t}}{4}$  can be rearranged into the form  $e^{3t} = \frac{e}{2\sqrt{(1 x)}}$ .
- (c) Hence find the Cartesian equation of C, giving your answer in the form

$$y = \frac{e}{f(x)[1 - \ln(f(x))]}$$

[2 marks]

[∠ mar
Answer space for question 7
$7a) x = 4 - e^{2-6t}$ $y = e^{3t}$
4 3t
$x = 1 - \frac{1}{4}e^{z-6t}$ $y = \frac{1}{3}t^{-1}e^{3t} \rightarrow u = \frac{1}{3}t^{-1}, v = e^{3t}$
$\frac{dx}{dt} = \frac{3}{3}e^{2-6t}$ $\frac{dy}{dt} = \frac{-1}{3}t^{-2}, dy = \frac{1}{3}t^{-2}$
$\frac{dt}{dt} = \frac{e^{3t} - e^{3t}}{3t^2} = \frac{3te^{3t} - e^{3t}}{3t^2}$
$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{dt}{dx}$
$= \frac{3t^2}{3t^2} \times \frac{3e^{2-6t}}{3}$
3t² 3e²-6t
$dy = 2(3te^{3t} - e^{3t})$
dx 9t2e2-6t
when $t=\frac{1}{3}$ , $dy = 2(3(\frac{1}{3})e^{3(\frac{1}{3})} - e^{3(\frac{1}{3})}$
The 9(2/3)22-6(4/3)
$= 2(2e^{2}-e^{2}) = 1e^{4}$ $4e^{-2}$
46-2



QUESTION PART REFERENCE	Answer space for question 7
6)	x = 4 - e <sup>2-6t</sup>
	4
	$4x = 4 - e^{z-6t}$
	e <sup>2-6t</sup> = 4 - 4x
	$e^{2} \times e^{-6t} = 4 - 4 \times$
	$e^2 = 4 - 4x$
	e 6+
	$e^{6t} = e^2 \left( \int \right)$
	4 - 4x
	Je6+ = e2
	4-4X
	e 3+ = e = e (as reg)
	5451-x 25(1-x)
	*,
_ ()	$y = e^{3t}$ $e^{3t} = e$ $2\sqrt{(1-x)}$ $3t \ln e = \ln\left(e\right)$ $2\sqrt{(1-x)}$
	3+ = 1/2 - 1/2 \(\int(1-x)\)
-	P
-	$y = \frac{2\sqrt{1-x}}{1-1/2\sqrt{1-x}}$ $\frac{3k-1-1/2\sqrt{1-x}}{1-1/2\sqrt{1-x}}$
	1-1/12 (1-)2)
	= e
	(2×5(1-x) (+-1/25(1-x))
	when (1x) = 25(1-x)
	Turn over ▶



- 8 It is given that  $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$ .
  - (a) By writing  $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$  as  $2\tan\theta = 3x$ , use implicit differentiation to show that  $\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{k}{4+9x^2}$ , where k is an integer.

[3 marks]

(b) Hence solve the differential equation

$$9y(4+9x^2)\frac{\mathrm{d}y}{\mathrm{d}x} = \csc 3y$$

given that x=0 when  $y=\frac{\pi}{3}$ . Give your answer in the form g(y)=h(x).

[7 marks]

QUESTION PART REFERENCE	Answer space for question 8	
8a)	$\theta = \tan^{-1}\left(\frac{3x}{2}\right)$	
		fon 0 = 32
	Itan D = 3x	
	Zsec 20 d0 = 3	1 2 x 4 3x
	dx	(1)
	$\frac{d\theta}{dx} = 3$	2
	dx 2 sec ' B	$\frac{1}{1}$ Sec $\theta = \frac{1}{2} = \frac{1}{2}$
		$Sec \theta = \frac{1}{600} = \frac{1}{2}$ $\sqrt{4+9x^2}$
1-1-1-10 V-1-1-10	d0 = 3	Sei 0 = 54+9x2
	$dx = 2\left(\frac{4+9x^2}{4}\right)$	7
	= 12	Sel 1 0 = 4 + 9x2
	$\frac{12}{2(4+9x^2)}$	4
	d0 = 6	
	$dx = 4 + 9x^2$	
	K=6	



Answer space for question 8
b) $9y(4+9x^2) dy = 6sec 3y$
$\int \frac{9y}{\cos 3y} dy = \int \frac{1}{4+9x^2} dx$
$\int \frac{9ij \sin 3y}{6} dy = 1 \int \frac{6}{4 + 9i^2} dx$
$\int \frac{9y \sin 3y}{6} dy = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
$-3y\cos 3y - \left(-3\cos 3y dy = \frac{1}{6}\tan^{-1}\left(3x\right) + c\right)$
- 3y cos 3y + \int 3 cos 3y dry = \frac{1}{6} \tan \cdot \left( \frac{3x}{2} \right) + C
- 3y (0) By + sin 3y = 1/6 tan' (3x) + C
sin 3y - 3y 613y = 1 tan'' (32) +c
$5c=0$ , when $y=\pi/3$ $5in(3(\pi/3))-3(\pi/3)\cos(3(\pi/3))=\frac{1}{6}tan^{-1}(0)+C$ $sin \pi - \pi \cos \pi = c$
$0 + \pi = C$ $50, \sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{11} \left(\frac{3x}{2}\right) + \pi$



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QUESTION PART REFERENCE	Answer space for question 8
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### **END OF QUESTIONS**

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